# A METHOD USING FOCAL PLANE ANALYSIS TO DETERMINE THE PERFORMANCE OF REFLECTOR ANTENNAS

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## **ABSTRACT**

Reflector antenna optimization schemes using array feeds and deformable flat plates (DFP) have been used to design systems that recover antenna losses resulting from antenna distortions. Historically, these optimization have been carried out using the antenna fir-field scattered patterns. The far-field patterns must be calculated separately for each of the antenna feed elements or for each different surface contour of the DFP. For large or complex antennas the far-field calculation times can be prohibitive.

This paper presents a method that allows the optimization to be canied out in the antenna focal region where the scattering calculation needs only to be done once independent of the number of the elements in the array or degrees of freedom in the DFP.

## 1. INTRODUCTION

At the NASA Deep Space Network (DSN) Goldstone Complex, located in the Mojave Desert in California, a 34-meter diameter beam waveguide (BWG) antenna, DSS-13, was constructed in 1988-1990 and has become an integral part of the advanced systems program and a test bed for technologies being developed to introduce Ka-band (32 GHz) frequencies into the DSN. antenna efficiency at 32 GHz was found to depend significantly on the elevation angle, i.e., it decreased from 45% to 35% as the elevation angle changed from 45 degrees to 20 degrees. This elevation angle dependence is due to the deformation in the main reflector caused by the resulting change in gravitational force applied to the antenna structure. Several methods, including using an anay feed and deformable flat plate, are under investigation to compensate for this distortion. This paper discusses the **reflector** analysis techniques used in the design of these systems.

1) Array feeds can be used to correct for the reflector distortions. Typical methods for optimizing the anay feed are very efficient when a fixed array geometry is utilized and only the feed excitation coefficients are optimized. For this case only one calculation of the radiating fields from each array element is required. For example, to maximize gain in a given direction, the

optimization can be as simple as taking the complex-conjugate of the secondary fields resulting from the illumination of the reflector in the given direction by each of the anay feed elements. For most existing methods, an optimization which allowed the element spacing and size to vary would be extremely time consuming since a radiation integral evaluation would be required for each feed element at each step of the optimization process.

A method of cornput ing anay feed performance is presented that obviates the need to recompute the reflector radiation fields when the feed clement size or spacing is varied. This technique is especially useful in a BWG system when the scattering calculation could require the cascading of as many as 6 mirrors. This therefore allows the optimization techniques to efficiently include size and spacing as parameters. While using focal plane analysis is not new (Ref. 1 for example), the application to beam waveguide antennas with its many mirrors and the derivation of a distorted surface from the analysis is unique.

The mathematical formulation is based upon the use of the Lorentz reciprocity theorem which convolves the focal plane distribution of the reflector system with the feed element aperture field distribution to obtain the element response. Thus the time consuming reflector system radiation integral evaluation is only done once for a given scan direction for all army feed geometry's considered.

2) A deformable flat plate (DFP) can also be used to correct for the gravity induced distortions. The same mathematical formulation can be used to both design the plate as well as **compute** the performance improvement. To design the plate, the fields of an incoming plane wave incident first upon the main reflector and then cascaded through the geometry to the flat plate from both a distorted and undistorted main reflector are computed. The phase differences between the two solutions arc translated via GO to surface deformation. The deformed surface thus obtained should compensate for the gain loss due to the main reflector distortion. Using the same incident fields (from the plane wave and distorted main reflector) onto the DFP and correlating with the fields from the feed horn, one can calculate the performance of the reflector system with the DFP installed.

Examples using the technique to design both an array feed and a deformable flat plate for the correction of gravity induced distortions of a large d(lal-shaped ground antenna are given.

#### 2. FOCAL PLANE ANALYSIS

The calculation of the gain of an antenna system by evacuating the fields in the antenna focal plane consists first of computing the focal fields produced by a plane wave impinging upon the aperture of the antenna. Second, the aperture fields of the focal horns located at the focal plane are determined, and these fields arc then convolved with the focal plane fields to provide the antenna gain. The process can be explained as follows. Consider a reflector antenna fed by a horn. We wish to determine the gain of this system in a given direction. (00, \$\Phi\_0\$), in the receive mode. First, consider the Lorentz reciprocity theorem

$$-\iint_{s} \left\{ \overline{E}_{a} \times \overline{H}_{b} - \overline{E}_{b} \times \overline{H}_{a} \right\} \cdot \overline{ds} =$$

$$\iiint_{s} \left\{ \overline{E}_{a} \cdot \overline{J}_{b} - \overline{H}_{a} \cdot \overline{M}_{b} - \overline{E}_{b} \cdot \overline{J}_{a} + \overline{H}_{b} \cdot \overline{M}_{a} \right\} dv$$
(1)

In this expression,  $\overline{E}_a$  and  $\overline{H}_a$  are fields radiated by a set of sources  $\overline{J}_a$  and  $\overline{M}_a$  and also  $\overline{E}_b$  and  $\overline{H}_b$  are fields radiated by sources  $\overline{J}_b$  and  $\overline{M}_b$ . The left integral is over a closed surface, which encloses the volume defined by the integral on the right side. Over an infinite region, the surface integral becomes zero. The Lorentz reciprocity theorem can therefore be rewritten as

$$\begin{split} & \iiint\limits_{v} \left\{ \overline{E}_{a} \cdot \overline{J}_{b} - \overline{H}_{a} \cdot \overline{M}_{b} \right\} dv = \\ & \iiint\limits_{v} \left\{ \overline{E}_{b} \cdot \overline{J}_{a} - \overline{H}_{b} \cdot \overline{M}_{a} \right\} dv \end{split} \tag{2}$$

which holds that as long as the relationship between the fields and their source currents holds true, the results will be the same wherever the integrals are evaluated in the region. Let us redefine the "a" sources as sources  $\overline{J}_{ha}$  and  $\overline{M}_{ha}$  to be associated with the feed horn apertures and generating the fields  $\overline{E}_{ha}$  and  $\overline{H}_{ha}$  in the aperture plane of the feed. In turn, let us redefine "b" sources as sources  $\overline{J}_{fp}$  and  $\overline{M}_{fp}$  to be associated with the antenna reflector system when illuminated by an incident plane-wave source from a direction  $(\theta_0, \phi_0)$ , and evaluated in the reflector system focal plane. The feed horn apertures are defined to be co-planar with the antenna focal plane.

Since the integration is limited to the aperture plane/focal plane, the integrals reduce to surface integrals. Each integral is proportional to the feed horn output voltage (Ref. 2). The left-hand equation is used since the program that generates the fbcxd-plane equivalent currents outputs currents and the program that computes the feed horn aperture distributions outputs fields. The expression relating the feed horn outputs  $v_r$ , to the currents from the antenna reflector system and the feed horn aperture fields is then

$$v_{r_{A}} \alpha \iint_{s} \left\{ \overline{E}_{ha} \cdot \overline{J}_{fp} - \overline{H}_{ha} \cdot \overline{M}_{fp} \right\} ds \tag{3}$$

The  $\overline{E}_{ha}$  and  $\overline{H}_{ha}$  should be determined in the presence of the antenna reflector system, and the focal plane currents  $\overline{J}_{fp}$  and  $\overline{M}_{fp}$  should be obtained when the feed horn is present. Such computations would require that the interactions between the feed horns and the reflectors be taken into consideration. Taking into account these interactions seriously complicates the analysis and increases the computational time. Often  $\overline{E}_{ha}$  and  $\overline{H}_{ha}$  are approximated to the aperture fields of a horn radiating into an infinite homogeneous free space (no reflector), and the focal-plane currents  $\overline{J}_{fp}$  and  $\overline{M}_{fp}$  of the antenna reflector system are also obtained in the absence of a feed. This is a reasonable assumption when the feed and antenna reflector system are widely separated in terms of wavelengths.

As will be seen later, there is also a **need** to obtain the **performance** of **a feed horn** in the presence **of a plane**-wave incident field arriving **from** a direction  $(\theta_p, \phi_p)$  and in the absence of the antenna reflector system. In the same manner as presented above, it can be shown that the output voltage for such a **feed** horn is

$$v_{p} \alpha \iint_{I} \left\{ \widetilde{E}_{ha} \cdot \overline{J}_{pw} - \overline{H}_{ha} \cdot \overline{M}_{pw} \right\} ds \tag{4}$$

where  $\overline{J}_{pw}$  and  $\overline{M}_{pw}$  are the currents in the feed aperture plane due to the incident plane wave field.

In Eqs. (3) and (4), the proportionality constants should be the same, being a function of the horn aperture characteristics. The proportionality constants can be eliminated by performing the ratio of Eqs. (3) and (4) as follows:

$$\frac{v_{r_{A}}}{v_{r_{P}}} = \frac{\iint \left\{ \overline{E}_{ha} \cdot \overline{J}_{fp} - \overline{H}_{ha} \cdot \overline{M}_{fp} \right\} ds}{\iint \left\{ \overline{E}_{ha} \cdot \overline{J}_{pw} - \overline{H}_{ha} \cdot \overline{M}_{pw} \right\} ds}$$
(5)

Let us now consider two transmit situations. First, let the feed horn radiate in the absence of the antenna reflectors and let the radiated field at  $(\tau, \theta_p, \phi_p)$  be  $F_a$ 

and the power be  $P_o$ . Then the gain of the feed horn in the direction  $(\theta_o, \phi_o)$  is

$$G_h = \frac{4\pi r^2}{\eta P_a} \left| E_h \right|^2 \tag{6}$$

Next, let the. horn illuminate the reflector system. The scattered field at ('r,  $\theta_o$ ,  $\phi_o$ ) is E.. Assume that the power that is radiated by the feed horn is still PO. Then the gain of the complete antenna system in the direction  $(\theta_o, \phi_o)$  is

$$G_a = \frac{4\pi r^2}{\eta P_a} |E_a|^2 \tag{7}$$

and, consequently,

$$G_{a} = G_{h} \frac{|E_{a}|^{2}}{|E_{h}|^{2}} \tag{8}$$

From reciprocity, we know that the radiated fields and horn output voltages are related by

$$\frac{\left|E_a\right|^2}{\left|E_h\right|^2} = \frac{\left|v_{r_A}\right|^2}{\left|v_{r_B}\right|^2} \tag{9}$$

Therefore, by combining Eqs (5) and (9), the overall gain of the reflector antenna system can be found in the receive mode from

$$G_{a} = G_{h} \left[ \frac{\iint \left\{ \overline{E}_{ha} \cdot \overline{J}_{fp} - \overline{H}_{ha} \cdot \overline{M}_{fp} \right\} ds |}{\iint \left\{ \overline{E}_{ha} \cdot \overline{J}_{fp} - \overline{H}_{ha} \cdot \overline{M}_{fp} \right\} ds |} \right]^{2}$$
(10)

It should be noted that  $(\theta_o, \phi_o)$  and  $(\theta_p, \phi_p)$  need not be the same. Therefore,  $(\theta_p, \phi_p)$  has been set to (0.0, 0.0) for simplicity of analysis when evaluating the interactions of the array feed with an incident plane wave and computing the feed horn far-field gain,  $G_b$ .

# 3. COMPENSATING REFLECTOR DISTOR-TIONS BY AN ARRAY FEED

As was mentioned in the introduction, the main purpose of the focal-plane optimization technique was to reduce the amount of computer time required to arrive at a solution. This is particularly true for studying large complex antennas such as the 34-meter beam-waveguide dual-shaped reflector antennas operating at 33.67 GHz that are used at the Jet Propulsion I aboratory/NASA deep space tracking network. In the conventional farfield optimization approach (Ref. 3), a full scattering calculation is required for each array feed element, for

cach antenna configuration, and for each array geometry. In the fbcd-plane analysis, a scattering calculation is required only for each antenna configuration number of array geometries and any number of feed elements can be studied without any further scattering calculations. With the large number of reflectors involved with beam-waveguide antennas and with the analysis being carried out at Ka-band, computation times of 6 hours or more arc common, thus limiting the number of scattering calculations for this type of applications is important. Although the ultimate application of this technique is for beam-waveguide antennas, to expedite the validation of the technique, the initial calculations were limited to the focal-plane of a dual-reflector antenna and these results are reported here. The goal was to compensate the 34-meter antenna for gain 10sscs that result from gravity-induced distortions as a function of the antenna's elevation angle where the surface is adjusted for minimum distortion at a 45degree elevation angle.

The case used for the elevation consisted of an array feed of seven equal sizedelements in a circular cluster on a triangular grid. The first step was to select an element size that minimized the antenna loss at one of the distortion extremes such as at an elevation angle of 7.5 degrees. A set of 13 element diameters were selected ranging from 2 cm to 13 cm. For each element diameter, the fbcal-plane optimization technique was used to determine the feed-clement weights that gave the best performance improvement. From the data it was found that an array of elements with diameters of 9.26 cm gave the best performance. Calculations using a conjugate match optimization technique were repeated at a series of antenna elevation angles using the 9.26-cmdiameter elements to determine the best performance that can be obtained based on the optimum element size determined at 7.5 degrees. Figure 1 summarizes the result. The first curve illustrates the performance that would be expected due to reflector distortions as a function of elevation angles if a single feed horn is used. The second curve shows the improvement in performance that can be obtained with the seven-clerncnt array using the optimum element weights computed by the optimization technique.

# 4. COMPENSATING REFLECTOR DISTORTIONS BY A DEFORMABLE FLATPLATE

Another technique for compensating gravity-induced structural deformations on the main reflector of a large aperture antenna is to utilize a deformable flat plate (DFP) installed at onc of the mirror locations in the beam waveguide optics [4]. Again, using the focal plane analysis provides an efficient way to compute the required DFP surface deformations.

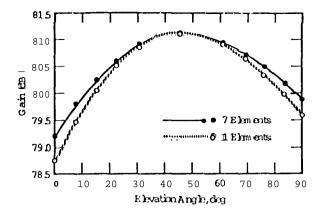


Figure 1: Optimum antenna gain vs. antenna elevation angle for 9.26-cm-diameter feed horn.

The procedure is as follows: 1) Compute the currents **induced on the flat mirror (without deformations) from the** undistorted main reflector due to a plane wave incident from the boresite direction, 2) Compute the currents induced on the flat plate from the **distorted** main reflector **from** the same plane wave, 3) Compute the phase difference between the two sets of currents on a point-by-point basis and 4) Use geometrical optics (GO) to convert the phase **differences** to **mechanical** displacements.

An initial low-cost demonstration of the technique was performed at a 34-meter beam waveguide (BWG) antenna using a fixed elevation angle and a manually adjustable DFP with 49 regularly spaced actuators The effective RMS surface error was improved from 0.59 mm for the initial no-correction flat plate to 0.49 mm for the initial analytically-derived correcting surface.

The experiment was repeated with a fully automated 16 actuator model.

The actuator locations were optimized **according** to **the** required contours, thus reducing the number of required actuators while maintaining the same level **of** performance.

#### 5. CONCLUSIONS

The focal plane method was shown to provide an efficient method for designing and analyzing the reflector performance for a large beam waveguide antenna utilizing either an array feed or a DFP to compensate for gravity induced distortions.

# 6. REFERENCES

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